Derivatives of Inverse Trig Functions

1) $y = \sin^{-1}(5x)$

$$y' = \frac{5}{\sqrt{1-(5x)^2}}$$

$$y' = \frac{5}{\sqrt{1-25x^2}}$$

2) $y = \csc^{-1}(4x^5)$

$$y' = \frac{-20x^4}{|4x^5|\sqrt{(4x^5)^2-1}}$$

3) $y = \arctan(e^{2x})$

$$y' = \frac{2e^{2x}}{1+(e^{2x})^2}$$

$$y' = \frac{2e^{2x}}{1+e^{4x}}$$

4) $y = \cot^{-1}(3x^2-1)$

$$y' = \frac{-6x}{1+(3x^2-1)^2}$$

5) $y = \arcsin\left(\frac{1}{x}\right)$

$$y' = \frac{-x^{-2}}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$$

$$y' = \frac{-\frac{1}{x^2}}{\sqrt{1-\frac{1}{x^2}}}$$

$$y' = \frac{-\frac{1}{x^2}}{\sqrt{\frac{x^2-1}{x^2}}}$$

$$y' = \frac{-\frac{1}{x^2}}{\frac{1}{x}\sqrt{x^2-1}}$$

$$y' = \frac{-1}{x\sqrt{x^2-1}}$$

6) $y = \text{arcsec}(x^3)$

$$y' = \frac{3x^2}{|x^3|\sqrt{(x^3)^2-1}}$$

7) $y = \arcsin x$; $x = \frac{\sqrt{2}}{2}$

<u>point</u>

$y\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

<u>slope</u>

$$y' = \frac{1}{\sqrt{1-x^2}} =$$

$$y'\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{\sqrt{1-\frac{1}{2}}} = \frac{1}{\sqrt{1/2}} = \sqrt{2}$$

Tangent: $y - \frac{\pi}{4} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right)$

8) $f(x) = \cos^{-1}(4x)$; $x = \frac{\sqrt{3}}{8}$

point

$f(\frac{\sqrt{3}}{8}) = \cos^{-1}(\frac{\sqrt{3}}{2})$

$= \frac{\pi}{6}$

$(\frac{\sqrt{3}}{8}, \frac{\pi}{6})$

slope

$f'(x) = \frac{-4}{\sqrt{1-(4x)^2}}$

$f'(\frac{\sqrt{3}}{8}) = \frac{-4}{\sqrt{1-(\frac{\sqrt{3}}{2})^2}}$

$f'(\frac{\sqrt{3}}{8}) = \frac{-4}{\sqrt{1/4}} = -8$

Tangent: $y - \frac{\pi}{6} = -8(x - \frac{\sqrt{3}}{8})$

9) $f(x) = \arctan x$; $x = 1$

point

$f(1) = \frac{\pi}{4}$

slope

$f'(x) = \frac{1}{1+x^2}$

$f'(1) = \frac{1}{2}$

Tangent: $y - \frac{\pi}{4} = \frac{1}{2}(x-1)$

10) $f(x) = \sin^{-1}(5x)$

point

$f(-\frac{\sqrt{3}}{10}) = \sin^{-1}(\frac{\sqrt{3}}{2})$

$= -\frac{\pi}{3}$

slope

$f'(x) = \frac{5}{\sqrt{1-(5x)^2}}$

$f'(-\frac{\sqrt{3}}{10}) = \frac{5}{\sqrt{1-(\sqrt{3}/2)^2}}$

$= \frac{5}{\sqrt{1/4}} = 10$

Tangent: $y + \frac{\pi}{3} = 10(x + \frac{\sqrt{3}}{10})$

11) $g(x) = (\arccos x^2)^5$

$g'(x) = 5\,\boxed{\arccos x^2} \cdot d\,\boxed{\arccos x^2}$

$= 5(\arccos x^2) \cdot \left[\frac{-2x}{\sqrt{1-x^4}}\right]$

12) $\lim_{x \to a} \frac{\arccos x - \arccos a}{x - a} = 3$

$\frac{-1}{\sqrt{1-a^2}} = 3$

$\frac{1}{1-a^2} = 9$

$1 - a^2 = \frac{1}{9}$

$-a^2 = -\frac{8}{9}$

$\boxed{a = \frac{\sqrt{8}}{3}}$

13) $\arctan y = \ln x$

$\frac{dy/dx}{1+y^2} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{1+y^2}{x}$

14) $y = e^x (\sec^{-1} x)$

$$\frac{dy}{dx} = (e^x)\left(\frac{1}{x\sqrt{x^2-1}}\right) + (\sec^{-1} x)(e^x)$$

15) $y^2 - 8y + x^2 = 5$

$$2y\frac{dy}{dx} - 8\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y-8}$$

$$\frac{dy}{dx} = \frac{-2}{y-4}$$